9

Probability

INTRODUCTION

In class XI, you have learnt about sample space, events, mutually exclusive and exhaustive events, independent and dependent events, law of total probability and Baye's theorem. In this chapter, you will learn the following:

(i) Probability distribution

(ii) Mathematical expectation and variance

(iii) Binomial distribution

(iv) Poisson distribution

(v) Normal distribution

RANDOM VARIABLES

A random variable is often described as a variable whose values are determined by chance i.e. by the outcomes of a random experiment.

If to each outcome of sample space we assign a real number, then we have a *function* defined on the sample space. It is called a *random function* or *stochastic function*. Loosely, it is called a *random variable* or *stochastic variable*. Thus, a *random variable is a function whose domain is the sample space of a random experiment and whose range is a subset of real numbers.*

For example, let S be the sample space of a simultaneous throw of two coins. Then, S = {HH, HT, TH, TT}.

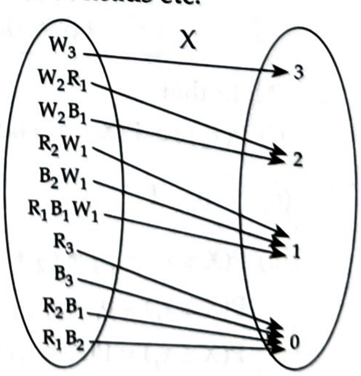
Let X denote the number of heads. Then X is a random variable. It can have values 0, 1 and 2 only. Thus, domain of X is S and range is {0, 1, 2}. Note that X(HT) = 1, X(TT) = 0 etc.

Let Y denote the number of tails. Then Y is also a random variable, having domain S and range {0, 1, 2}.

Note that though domain and range of X and Y is same, $X \neq Y$ because X(HH) = 2 while Y(HH) = 0. Also note that many other random variables can be defined over this sample space; for example, number of heads minus number of tails, square of the number of heads etc.

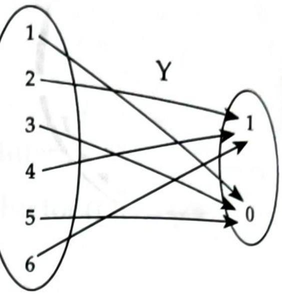
Some other examples of random variables are :

(i) Three balls are drawn simultaneously from a bag containing 5 red, 3 black and 4 white balls. Let X be a variable defined as the number of white balls drawn. Then X is a random variable with domain = sample space S = {W_3, R_3, B_3, W_2R_1, W_2B_1, R_2W_1, R_2B_1, B_2R_1, B_2W_1, $B_1R_1W_1$} where W_1, W_2, W_3 denote one, two, three white balls respectively etc. and range = {0, 1, 2, 3}.



Note that $X(W_3) = 3$, $X(W_2 B_1) = 2$, $X(R_2 B_1) = 0$, $X(R_3) = 0$ etc. The random function X is shown in the diagram given on the previous page.

(*ii*) A die is rolled. Let "success" (value = 1) be defined as getting an even number and "failure" (value = 0) be defined as getting an odd number. The random function Y associated with this experiment is shown in the adjoining diagram. The domain is {1, 2, 3, ..., 6} and range is {0, 1}.



A random variable is called **discrete** if it assumes only *finite or countably infinite* number of values; for example, the number of heads in n tosses of a coin. The random variable which can assume non-countably infinite number of values is called **non-discrete** or **continuous random variable.** It can assume values in an interval. For example, height of a person in a group of students.

PROBABILITY DISTRIBUTION OF RANDOM VARIABLES

As mentioned earlier, when two coins are tossed (or a coin is tossed twice), the sample space is S = {HH, HT, TH, TT}. If a random variable X is defined on this sample space as the number of heads obtained, then X can assume values 0, 1, 2 only. Probability that X assumes value 0 is

$$P(X = 0) = P(TT) = \frac{1}{4}.$$

$$\text{Similarly, } P(X = 1) = P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2},$$

$$P(X = 2) = P(HH) = \frac{1}{4}.$$

You know that in a frequency distribution or frequency table, the values of frequencies are written against the values of the variate. Similarly, in a probability distribution, the probabilities corresponding to all possible values of the random variable are written against the value of the random variable. Thus, in the above example, we get the following probability distribution :

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Note that $\Sigma P(X) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$, and $P(X) \geq 0$ for all values of X. Also observe that while random variables are denoted by capital letters X, Y, ... their values are denoted by small letters $x, y, \ldots$

More generally, if X is a discrete random variable which can assume values $x_1, x_2, x_3, \ldots, x_n$ (arranged in increasing order of magnitude) with probabilities $p_1, p_2, p_3, \ldots, p_n$, then the probability distribution can be represented in tabular form as

X	x_1	x_2	...	x_n
$P(X = x)$	p_1	p_2	...	p_n

Note that

(*i*) $P(x_k)$ *i.e.* $P(X = x_k)$ lies between 0 and 1 *i.e.* $0 \leq P(x_k) \leq 1$ for $k = 1, 2, \ldots, n$

(*ii*) $\sum_{i=1}^{n} p_i = 1$

(*iii*) $P(X \leq x_i) = p_1 + p_2 + \ldots + p_i$,

$P(X \geq x_i) = p_i + p_{i+1} + \ldots + p_n$,

$P(X \leq x_i) = P(X < x_i) + P(X = x_i)$ etc.

ILLUSTRATIVE EXAMPLES

Example 1. *State whether it is possible for a random variable to have any of the following probability distributions :*

(*i*)

X	0	1	2
P(X)	0.4	0.4	0.2

(*ii*)

Y	1	2	3	5
P(Y)	0.2	0.3	0.2	0.2

(*iii*)

Z	1	2	3	4	5
P(Z)	0.1	0.4	0.05	−0.2	0.2

Solution.

(*i*) We observe that $0 \le P(X) \le 1$ for all X and $\Sigma P(X) = 0.4 + 0.4 + 0.2 = 1$.
Hence, we have a valid probability distribution.

(*ii*) We observe that though $0 \le P(Y) \le 1$ for all Y, $\Sigma P(Y) = 0.2 + 0.3 + 0.2 + 0.2 = 0.9 \neq 1$. Hence, it is not possible for a random variable to have this probability distribution.

(*iii*) We observe that $P(4) = -0.2 < 0$. Hence, it is not possible for a random variable to have this probability distribution.

Example 2. *A random variable X has the following probability distribution :*

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine :

(*i*) k (*ii*) $P(X < 3)$ (*iii*) $P(X > 6)$ (*iv*) $P(0 < X < 3)$.

Solution. (*i*) We know that $\Sigma p_i = 1$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow (10k - 1)(k + 1) = 0$$

$$\Rightarrow k = \frac{1}{10}, -1 \text{ but } k \text{ cannot be negative}$$

$$\Rightarrow k = \frac{1}{10}.$$

(*ii*) $P(X < 3) = P(0) + P(1) + P(2) = 0 + k + 2k$

$$= 3k = 3 \times \frac{1}{10} = \frac{3}{10}.$$

(*iii*) $P(X > 6) = P(7) = 7k^2 + k = 7 \cdot \frac{1}{100} + \frac{1}{10} = \frac{17}{100}.$

(*iv*) $P(0 < X < 3) = P(1) + P(2) = k + 2k = 3k = 3 \times \frac{1}{10} = \frac{3}{10}.$

Example 3. *Let X denote the number of hours you study on a Sunday. Also it is known that*

$$P(X = x) = \begin{cases} 0.1 & , \text{ if } x = 0 \\ kx & , \text{ if } x = 1 \text{ or } 2 \\ k(5 - x) & , \text{ if } x = 3 \text{ or } 4 \\ 0 & , \text{ otherwise} \end{cases}$$

where k is a constant.

(i) *Find the value of k.*

(ii) *What is the probability that you study atleast two hours? Exactly two hours? Atmost two hours?*

Solution. From given information, we find that the probability distribution of X is

X	0	1	2	3	4
P(X)	0.1	k	$2k$	$2k$	k

(i) We know that $\Sigma p_i = 1$

$\Rightarrow$ $0.1 + k + 2k + 2k + k = 1 \Rightarrow 6k = 1 - 0.1 = 0.9$

$\Rightarrow$ $k = 0.15$

(ii) $\text{P(you study atleast two hours)} = P(X \geq 2)$

$= P(2) + P(3) + P(4) = 2k + 2k + k$

$= 5k = 5 \times 0.15 = 0.75$

$\text{P(you study exactly two hours)} = P(2) = 2k = 2 \times 0.15 = 0.3$

$\text{P(you study atmost two hours)} = P(X \leq 2) = P(0) + P(1) + P(2)$

$= 0.1 + k + 2k = 0.1 + 3k$

$= 0.1 + 3 \times 0.15 = 0.55$

Example 4. *Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that*

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).$$

Find the probability distribution of X.

Solution. Given $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4) = k$ (say)

$\Rightarrow$ $P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}, P(X = x_3) = k, P(X = x_4) = \frac{k}{5}.$

We know that $\Sigma P(X = x_i) = 1 \Rightarrow \frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1$

$\Rightarrow$ $\left(\frac{1}{2} + \frac{1}{3} + 1 + \frac{1}{5}\right)k = 1 \Rightarrow \frac{15 + 10 + 30 + 6}{30} k = 1$

$\Rightarrow$ $\frac{61}{30} k = 1 \Rightarrow k = \frac{30}{61}.$

$\therefore$ $\frac{k}{2} = \frac{15}{61}, \frac{k}{3} = \frac{10}{61}, \frac{k}{5} = \frac{6}{61}.$

$\therefore$ The probability distribution of X = $\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ \frac{15}{61} & \frac{10}{61} & \frac{30}{61} & \frac{6}{61} \end{pmatrix}$

Example 5. *The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3)$ such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p.*

Solution. Given $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3) = k$ (say)

The probability distribution of the random variable X is

X	0	1	2	3
P(X)	p	p	k	k

We know that $\Sigma p_i = 1$

$\Rightarrow$ $p + p + k + k = 1 \Rightarrow 2p + 2k = 1$

$\Rightarrow$ $p + k = \frac{1}{2} \Rightarrow k = \frac{1}{2} - p.$

We construct the following table:

x_i	p_i	p_ix_i	$p_ix_i^2$
0	p	0	0
1	p	p	p
2	$\frac{1}{2}-p$	$1-2p$	$2-4p$
3	$\frac{1}{2}-p$	$\frac{3}{2}-3p$	$\frac{9}{2}-9p$
Total		$\frac{5}{2}-4p$	$\frac{13}{2}-12p$

Given $\Sigma p_ix_i^2 = 2\Sigma p_ix_i$

$$\Rightarrow \quad \frac{13}{2}-12p = 2\left(\frac{5}{2}-4p\right) \Rightarrow \frac{13}{2}-12p = 5-8p \Rightarrow -4p = 5-\frac{13}{2}$$

$$\Rightarrow \quad -4p = -\frac{3}{2} \Rightarrow p = \frac{3}{8}.$$

Hence, the value of p is $\frac{3}{8}$.

Example 6. *Find the probability distribution of X, the number of heads in three tosses of a coin (or a simultaneous toss of three coins).*

Solution. The sample space associated with this experiment is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

All 8 outcomes are equally likely.

Let random variable X be defined as the number of heads, then X can take the values 0, 1, 2 and 3.

$$P(X = 0) = P(TTT) = \frac{1}{8}$$

$$P(X = 1) = P(TTH \text{ or } THT \text{ or } HTT) = \frac{3}{8}$$

$$P(X = 2) = P(HHT \text{ or } HTH \text{ or } THH) = \frac{3}{8}$$

$$P(X = 3) = P(HHH) = \frac{1}{8}.$$

Hence, the required probability distribution is

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

We may also write it as $\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$.

Example 7. *A person plays a game of tossing a coin thrice. For each head he gets ₹2 from the organiser, and for each tail he has to give ₹1.50 to the organiser. Let X denote the amount gained or lost by the person. Show that X is a random variable and exhibit it as a function on the sample space.*

Solution. X is a number whose values are defined on the outcomes of a random experiment. Hence X is, by definition, a random variable.

Now, the sample space of the experiment is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Also,

$$X(HHH) = ₹(2 \times 3) = ₹6,$$
$$X(HHT) = X(HTH) = X(THH) = ₹(2 \times 2 - 1 \times 1.50) = ₹2.50$$
$$X(HTT) = X(THT) = X(TTH) = ₹(1 \times 2 - 2 \times 1.50) = -₹1$$
$$X(TTT) = ₹(3 \times (-1.50)) = -₹4.50$$

Minus sign shows loss to the player.

Thus, for each element of the sample space, X takes a unique value. Hence, X is a function on the sample space S, and its range is $\{-4.50, -1, 2.50, 6\}$.

Example 8. *A coin is tossed until a head appears or the tail appears 4 times in succession. Find the probability distribution of the number of tosses.*

Solution. The sample space associated with this experiment is

$$S = \{H, TH, TTH, TTTH, TTTT\}$$

Let the random variable X be defined as the number of tosses. Then X can take the values 1, 2, 3, 4. The corresponding probabilities are

$$P(X = 1) = P(H) = \frac{1}{2}$$

$$P(X = 2) = P(TH) = \frac{1}{4}$$

$$P(X = 3) = P(TTH) = \frac{1}{8}$$

$$P(X = 4) = P(TTTH, TTTT) = \frac{1}{16} + \frac{1}{16} = \frac{1}{8}.$$

We observe that $\Sigma P(X) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = 1.$

The probability distribution of the number of tosses is

X	1	2	3	4
P(X)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

or simply, $\begin{pmatrix} 1 & 2 & 3 & 4 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

Example 9. *Two cards are drawn simultaneously from a well-shuffled deck of 52 playing cards. Find the probability distribution of number of aces.*

Solution. Let the random variable X be defined as the number of aces in a draw of 2 cards simultaneous, then X can take the values 0, 1, 2. The corresponding probabilities are:

$$P(X = 0) = P(\text{drawing no ace}) = P(\text{drawing both non-ace cards})$$
$$= \frac{^{48}C_2}{^{52}C_2} = \frac{48 \times 47}{1 \times 2} \times \frac{1 \times 2}{52 \times 51} = \frac{188}{221},$$

$$P(X = 1) = P(\text{drawing one ace and one non-ace card})$$
$$= \frac{^{4}C_1 \times {^{48}C_1}}{^{52}C_2} = \frac{4}{1} \times \frac{48}{1} \times \frac{1 \times 2}{52 \times 51} = \frac{32}{221}.$$

$$P(X = 2) = P(\text{drawing both aces}) = \frac{^{4}C_2}{^{52}C_2} = \frac{4 \times 3}{1 \times 2} \times \frac{1 \times 2}{52 \times 51} = \frac{1}{221}.$$

The probability distribution of number of aces is :

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Example 10. *From a lot of 10 bulbs, which includes 3 defective bulbs, a sample of 2 bulbs is drawn at random. Find the probability distribution of defective bulbs.*

Solution. Total number of bulbs = 10,

number of defective bulbs = 3,

$\therefore$ number of good bulbs = 10 – 3 = 7.

Two bulbs can be drawn (simultaneously) in ${}^{10}C_2$ ways.

Let X denote the number of defective bulbs in a draw of 2 bulbs, then X can take the values 0, 1, 2.

P(X = 0) = P(drawing no defective bulb) = P(drawing 2 good bulbs)

$$= \frac{{}^7C_2}{{}^{10}C_2} = \frac{7\times 6}{1\times 2}\times\frac{1\times 2}{10\times 9} = \frac{7}{15}.$$

P(X = 1) = P(drawing one defective bulb and one good bulb)

$$= \frac{{}^3C_1\times {}^7C_1}{{}^{10}C_2} = \frac{3}{1}\times\frac{7}{1}\times\frac{1\times 2}{10\times 9} = \frac{7}{15}.$$

P(X = 2) = P(drawing two defective bulbs)

$$= \frac{{}^3C_2}{{}^{10}C_2} = \frac{3\times 2}{1\times 2}\times\frac{1\times 2}{10\times 9} = \frac{1}{15}.$$

$\therefore$ The probability distribution of defective bulbs is :

X	0	1	2
P(X)	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

Example 11. *Two dice are rolled simultaneously. Find the probability distribution of sum of digits on the two dice.*

Solution. The sample space associated with this experiment is

S = {(1, 1), (1, 2), (1, 3), ..., (1, 6), (2, 1), (2, 2) ..., (2, 6), (3, 1), ..., (3, 6), (4, 1), ..., (4, 6), (5, 1), ..., (5, 6), (6, 1), ..., (6, 6)}.

All 36 outcomes are equally likely.

Let random variable X be defined as the sum of digits on the two dice, then X can take the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

$$P(X = 2) = P\{(1, 1)\} = \frac{1}{36}$$

$$P(X = 3) = P\{(1, 2), (2, 1)\} = \frac{2}{36} = \frac{1}{18}$$

$$P(X = 4) = P\{(1, 3), (3, 1), (2, 2)\} = \frac{3}{36} = \frac{1}{12}$$

$$P(X = 5) = P\{(1, 4), (4, 1), (2, 3), (3, 2)\} = \frac{4}{36} = \frac{1}{9}$$

$$P(X = 6) = P\{(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)\} = \frac{5}{36}$$

$$P(X = 7) = P\{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\} = \frac{6}{36} = \frac{1}{6}$$

$$P(X = 8) = P\{(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)\} = \frac{5}{36}$$

$$P(X = 9) = P\{(3, 6), (6, 3), (4, 5), (5, 4)\} = \frac{4}{36} = \frac{1}{9}$$

$$P(X = 10) = P\{(4, 6), (6, 4), (5, 5)\} = \frac{3}{36} = \frac{1}{12}$$

$$P(X = 11) = P\{(5, 6), (6, 5)\} = \frac{2}{36} = \frac{1}{18}$$

$$P(X = 12) = P\{(6, 6)\} = \frac{1}{36}.$$

Hence, the required probability distribution is

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

EXERCISE 9.1

1. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	a	$4a$	$3a$	$7a$	$8a$	$10a$	$6a$	$9a$

   (*i*) Determine the value of a.

   (*ii*) Find $P(X < 3)$, $P(X \geq 4)$, $P(0 < X < 5)$.

2. A random variable X has the probability distribution:

X	1	2	3	4	5	6	7	8
P(X)	0.15	0.23	0.12	0.1	0.2	0.08	0.07	0.05

   If events E = {X is a prime number} and F = {X < 4}, then find $P(E \cup F)$.

   Hint. $E \cup F = \{1, 2, 3, 5, 7\}$

   $\Rightarrow \quad P(E \cup F) = P(1) + P(2) + P(3) + P(5) + P(7).$

3. A random variable X has the following probability distribution where k is some number:

$$P(x) = \begin{cases} k & , \text{if } x = 0 \\ 2k & , \text{if } x = 1 \\ 3k & , \text{if } x = 2 \\ 0 & , \text{otherwise} \end{cases}$$

   (*i*) Determine the value of k.

   (*ii*) Find $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$.

4. A coin is tossed twice (or two coins are tossed simultaneously).
   (i) Find the probability distribution of X, the number of heads.
   (ii) If a random variable Y is defined on this sample space as the number of tails, then find the probability distribution of Y.
5. A die is rolled. If a random variable X is defined as the number on the upper face, then find its probability distribution.
6. An urn contains 5 white and 3 black balls. If two balls are drawn at random without replacement and X denotes the number of white balls drawn, then find its probability distribution.
7. Two cards are drawn simultaneously from a well-shuffled pack of 52 cards. Find the probability distribution of the number of jacks.
8. Four bad oranges are mixed accidentally with 16 good oranges. Find the probability distribution of the number of bad oranges in a draw of two oranges.
9. Three balls are drawn one by one without replacement from a bag containing 5 white and 4 red balls. Find the probability distribution of the number of white balls drawn.
10. A box contains 12 bulbs of which 3 are defective. A sample of 3 bulbs is selected from the box. Let X denote the number of defective bulbs in the sample. Find the probability distribution of X.
11. Four bad eggs are mixed with 10 good ones. If 3 eggs are drawn one by one without replacement, then find the probability distribution of the number of bad eggs drawn.

Answers

1. (i) $\frac{1}{48}$ (ii) $P(X < 3) = \frac{1}{6}, P(X \geq 4) = \frac{33}{48}, P(0 < X < 5) = \frac{11}{24}$

2. 0.77

3. (i) $\frac{1}{6}$ (ii) $\frac{1}{2}, 1, \frac{1}{2}$

4. (i) $\begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$

5. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

6. $\begin{pmatrix} 0 & 1 & 2 \\ \frac{3}{28} & \frac{15}{28} & \frac{5}{14} \end{pmatrix}$

7. $\begin{pmatrix} 0 & 1 & 2 \\ \frac{188}{221} & \frac{32}{221} & \frac{1}{221} \end{pmatrix}$

8. $\begin{pmatrix} 0 & 1 & 2 \\ \frac{12}{19} & \frac{32}{95} & \frac{3}{95} \end{pmatrix}$

9. $\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{21} & \frac{5}{14} & \frac{10}{21} & \frac{5}{42} \end{pmatrix}$

10.

X	0	1	2	3
P(X)	$\frac{84}{220}$	$\frac{108}{220}$	$\frac{27}{220}$	$\frac{1}{220}$

11. $\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{30}{91} & \frac{45}{91} & \frac{15}{91} & \frac{1}{91} \end{pmatrix}$

MEAN AND VARIANCE OF PROBABILITY DISTRIBUTION

You must have noted the striking resemblance between probability distributions and frequency distributions. In frequency distribution, we define (arithmetic) mean as

$$\bar{x} = \mu = \frac{\Sigma f_i x_i}{\Sigma f_i}.$$

Similarly, in a discrete probability distribution, we define mean as

$$\mu = \frac{\Sigma p_i x_i}{\Sigma p_i}$$

As $\quad \Sigma p_i = 1$, we get

$$\textbf{Mean } \mu = \Sigma\, p_i\, x_i = p_1x_1 + p_2x_2 + \dots + p_nx_n$$

Mean is also called ***average*** or ***expected value*** or ***expectation***, denoted as E(X).

Similarly, the variance denoted by Var(X) or σ_x^2 or σ^2 is defined as

$$\sigma^2 = \Sigma p_i(x_i - \mu)^2$$

As
$$\Sigma p_i\,(x_i - \mu)^2 = \Sigma p_i\,(x_i^2 - 2\mu x_i + \mu^2)$$
$$= \Sigma p_i x_i^2 - 2\mu\,\Sigma p_i x_i + \mu^2 \Sigma p_i$$
$$= \Sigma p_i x_i^2 - 2\mu.\mu + \mu^2.1 = \Sigma p_i x_i^2 - \mu^2$$

Thus,
$$\sigma^2 = \Sigma p_i(x_i - \mu)^2 = \Sigma p_i x_i^2 - \mu^2 = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$$

The standard deviation is

$$\sigma = \sqrt{\Sigma\, p_i\, x_i^2 - \mu^2}$$

Note that $\quad \sigma = \sqrt{E(X^2) - (E(X))^2}$, where $E(X^2) = \Sigma p_i x_i^2$.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the mean, variance and standard deviation of the probability distribution :*

X	2	3	4
P(X)	0.2	0.5	0.3

Solution. We construct the following table :

x_i	p_i	$p_i x_i$	$p_i x_i^2$
2	0.2	0.4	0.8
3	0.5	1.5	4.5
4	0.3	1.2	4.8
Total		3.1	10.1

$\therefore$ Mean = μ = 3.1

Variance = $\Sigma\, p_i\, x_i^2 - \mu^2 = 10.1 - (3.1)^2 = 10.1 - 9.61 = 0.49$

Standard deviation = $\sqrt{0.49} = 0.7$

Example 2. *A random variable X has the following probability distribution :*

X	0	1	2	3	4	5	6
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2 + C$

Find the value of C and also calculate the mean of the distribution.

Solution. We know that $\Sigma p_i = 1$

$\Rightarrow$ $C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$

$\Rightarrow$ $10C^2 + 9C - 1 = 0$

$\Rightarrow$ $(10C - 1)(C + 1) = 0$

$\Rightarrow$ $C = \frac{1}{10}$ or $C = -1$ but probability cannot be negative

$\Rightarrow$ $C = \frac{1}{10}$.

$$\text{Mean} = \Sigma p_i x_i = C \times 0 + 2C \times 1 + 2C \times 2 + 3C \times 3 + C^2 \times 4 + 2C^2 \times 5 + (7C^2 + C) \times 6$$

$$= 56C^2 + 21C = 56 \times \frac{1}{100} + 21 \times \frac{1}{10} = \frac{266}{100} = \frac{133}{50}.$$

Example 3. *What is the mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face?* (CBSE 2022 Term I)

Solution. Given on three faces of a die 1 is written, on two faces 2 is written and on one face 5 is written.

The die is thrown once. The sample space of the random experiment = {1, 1, 1, 2, 2, 5}. It has six equally likely outcomes.

$\therefore$ $P(1) = \frac{3}{6} = \frac{1}{2}$, $P(2) = \frac{2}{6} = \frac{1}{3}$ and $P(5) = \frac{1}{6}$.

$\therefore$ Probability distribution of the numbers written on face of a die :

x_i	1	2	5
$P(x_i)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$$\text{Mean} = \Sigma p_i x_i = \frac{1}{2} \times 1 + \frac{1}{3} \times 2 + \frac{1}{6} \times 5 = \frac{1}{2} + \frac{2}{3} + \frac{5}{6}$$

$$= \frac{3 + 4 + 5}{6} = \frac{12}{6} = 2.$$

Example 4. *Find the mean, variance and standard deviation of the number of tails in three tosses of a coin.*

Solution. When a coin is tossed three times, then the sample space S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.

Let random variable X denote the number of tails, so X can take values 0, 1, 2, 3.

$$P(X = 0) = P(\text{no tails}) = P(\text{HHH}) = \frac{1}{8},$$

$$P(X = 1) = P(\text{one tail}) = P(\text{HHT, HTH, THH}) = \frac{3}{8},$$

$$P(X = 2) = P(\text{two tails}) = P(\text{HTT, THT, TTH}) = \frac{3}{8},$$

$$P(X = 3) = P(\text{three tails}) = P(\text{TTT}) = \frac{1}{8}.$$

We construct the following table :

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
Total		$\frac{12}{8} = \frac{3}{2}$	$\frac{24}{8} = 3$

$\therefore$ Mean $= \mu = \Sigma\, p_i\, x_i = \frac{3}{2}$.

Variance $= \sigma^2 = \Sigma\, p_i\, x_i^2 - \mu^2 = 3 - \left(\frac{3}{2}\right)^2 = 3 - \frac{9}{4} = \frac{3}{4}$.

Standard deviation $= \sigma = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$.

Example 5. *In a manufacturing unit inspection from a lot of 20 baskets which include 6 defectives, a sample of 2 baskets is drawn at random without replacement. Prepare the probability distribution of the number of defective baskets. Also, calculate E(X) for the random variable X.* (CBSE 2022 Term I)

Solution. Let the random variable X be defined as the number of defective baskets in a sample of two baskets, then X can take values 0, 1, 2.

Total number of baskets = 20.

Two baskets can be drawn simultaneously in ${}^{20}C_2$ ways

$$P(X = 0) = P(\text{drawing no defective basket})$$

$$= \frac{{}^{14}C_2}{{}^{20}C_2} = \frac{14 \times 13}{20 \times 19} = \frac{91}{190}$$

$$P(X = 1) = P(\text{drawing one defective basket})$$

$$= \frac{{}^{14}C_1 \times {}^{6}C_1}{{}^{20}C_2} = \frac{2 \times 14 \times 6}{20 \times 19} = \frac{42}{95}$$

$$P(X = 2) = P(\text{drawing two defective baskets})$$

$$= \frac{{}^{6}C_2}{{}^{20}C_2} = \frac{6 \times 5}{20 \times 19} = \frac{3}{38}.$$

Now, $E(X) = \Sigma p_i x_i$

$$= 0 \times \frac{91}{190} + 1 \times \frac{42}{95} + 2 \times \frac{3}{38}$$

$$= \frac{84 + 30}{190} = \frac{114}{190} = 0.6$$

Example 6. *Four bad oranges are accidently mixed with sixteen good oranges. Find the probability distribution of the number of bad oranges in a draw of two oranges. Also find the mean, variance and standard deviation of the distribution.*

Solution. Let the random variable X be defined as the number of bad oranges in a draw of two oranges. Then X can take values 0, 1, 2.

Total number of oranges = 4 + 16 = 20.

Two oranges can be drawn (simultaneously) in ${}^{20}C_2$ ways.

$$P(X = 0) = P(\text{drawing no defective orange}) = P(\text{both good oranges})$$

$$= \frac{{}^{16}C_2}{{}^{20}C_2} = \frac{16 \times 15}{1 \times 2} \times \frac{1 \times 2}{20 \times 19} = \frac{12}{19},$$

$$P(X = 1) = P(\text{drawing one defective orange and one good orange})$$

$$= \frac{{}^{4}C_1 \times {}^{16}C_1}{{}^{20}C_2} = \frac{4}{1} \times \frac{16}{1} \times \frac{1 \times 2}{20 \times 19} = \frac{32}{95},$$

$$P(X = 2) = P(\text{drawing two defective oranges})$$

$$= \frac{{}^{4}C_2}{{}^{20}C_2} = \frac{4 \times 3}{1 \times 2} \times \frac{1 \times 2}{20 \times 19} = \frac{3}{95}.$$

$\therefore$ The probability distribution is $\begin{pmatrix} 0 & 1 & 2 \\ \frac{12}{19} & \frac{32}{95} & \frac{3}{95} \end{pmatrix}$.

To calculate the mean and variance of the distribution, we construct the following table:

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{12}{19}$	0	0
1	$\frac{32}{95}$	$\frac{32}{95}$	$\frac{32}{95}$
2	$\frac{3}{95}$	$\frac{6}{95}$	$\frac{12}{95}$
Total		$\frac{38}{95} = \frac{2}{5}$	$\frac{44}{95}$

$\therefore$ Mean, $\mu = \Sigma p_i x_i = \frac{2}{5}$.

Variance, $\sigma^2 = \Sigma p_i x_i^2 - \mu^2 = \frac{44}{95} - \left(\frac{2}{5}\right)^2 = \frac{44}{95} - \frac{4}{25} = \frac{144}{475}$.

Standard deviation $= \sqrt{\frac{144}{475}} = \frac{12}{5\sqrt{19}}$.

Example 7. *A class XII has 20 students whose marks (out of 30) are 14, 17, 25, 14, 21, 17, 17, 19, 18, 26, 18, 17, 17, 26, 19, 21, 21, 25, 14 and 19. If random variable X denotes the marks of a selected student given that the probability of each student to be selected is equally likely.*

(a) Prepare the probability distribution of the random variable X.

(b) Find mean, variance and standard deviation of X.

Solution. (*a*) Let us prepare the following frequency table:

Marks obtained	14	17	18	19	21	25	26
No. of students	3	5	2	3	3	2	2

Total number of students = 20.

Given that X = marks of a selected student.

So, $P(X = 14) = \frac{3}{20}$; $P(X = 17) = \frac{5}{20} = \frac{1}{4}$; $P(X = 18) = \frac{2}{20} = \frac{1}{10}$;

$P(X = 19) = \frac{3}{20}$; $P(X = 21) = \frac{3}{20}$; $P(X = 25) = \frac{2}{20} = \frac{1}{10}$;

$P(X = 26) = \frac{2}{20} = \frac{1}{10}$.

Hence, the required probability distribution is

X	14	17	18	19	21	25	26
P(X)	$\frac{3}{20}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{3}{20}$	$\frac{3}{20}$	$\frac{1}{10}$	$\frac{1}{10}$

(*b*) To calculate mean, variance and standard deviation, we construct the following table:

x_i	p_i	$p_i x_i$	$p_i x_i^2$
14	$\frac{3}{20}$	$\frac{42}{20}$	$\frac{147}{5}$
17	$\frac{1}{4}$	$\frac{17}{4}$	$\frac{289}{4}$
18	$\frac{1}{10}$	$\frac{18}{10}$	$\frac{162}{5}$
19	$\frac{3}{20}$	$\frac{57}{20}$	$\frac{1083}{20}$
21	$\frac{3}{20}$	$\frac{63}{20}$	$\frac{1323}{20}$
25	$\frac{1}{10}$	$\frac{25}{10}$	$\frac{125}{2}$
26	$\frac{1}{10}$	$\frac{26}{10}$	$\frac{338}{5}$
Total		$\frac{385}{20}$	$\frac{7689}{20}$

$\therefore$ Mean $\mu = \Sigma p_i x_i = \frac{385}{20} = \frac{77}{4} = 19.25$

$$\text{Variance } \sigma^2 = \Sigma p_i x_i^2 - \mu^2 = \frac{7689}{20} - \left(\frac{77}{4}\right)^2 = \frac{7689}{20} - \frac{5929}{16}$$

$$= \frac{30756 - 29645}{80} = \frac{1111}{80} = 13.89$$

Standard deviation $\sigma = \sqrt{\text{Variance}} = \sqrt{13.89} = 3.73$

Example 8. *Two cards are drawn simultaneously from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.*

Solution. Let random variable X denote the number of red cards drawn in a draw of 2 cards from a pack of 52 cards, then X can take values 0, 1, 2.

$$P(X = 0) = P(\text{2 black cards}) = \frac{^{26}C_2}{^{52}C_2} = \frac{26 \times 25}{1 \times 2} \times \frac{1 \times 2}{52 \times 51} = \frac{25}{102},$$

$$P(X = 1) = P(\text{1 red card and 1 black card}) = \frac{^{26}C_1 \times {^{26}C_1}}{^{52}C_2} = \frac{26 \times 26 \times 1 \times 2}{52 \times 51} = \frac{52}{102},$$

$$P(X = 2) = P(\text{2 red cards}) = \frac{^{26}C_2}{^{52}C_2} = \frac{26 \times 25}{1 \times 2} \times \frac{1 \times 2}{52 \times 51} = \frac{25}{102}.$$

We construct the table as under :

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{25}{102}$	0	0
1	$\frac{52}{102}$	$\frac{52}{102}$	$\frac{52}{102}$
2	$\frac{25}{102}$	$\frac{50}{102}$	$\frac{100}{102}$
Total		$\frac{102}{102} = 1$	$\frac{152}{102}$

$\therefore$ Mean $= \Sigma p_i x_i = 1.$

$$\text{Variance} = \Sigma p_i x_i^2 - \mu^2 = \frac{152}{102} - (1)^2 = \frac{50}{102} = \frac{25}{51}.$$

Example 9. *Let X denote the number of hours a person watches television during a randomly selected day. The probability that X can take the values x_i has the following form, where k is some unknown constant.*

$$P(X = x_i) = \begin{cases} 0.2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2. \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) *Find the value of k.*

(b) *What is the probability that the person watches two hours of television on a selected day?*

(c) *What is the probability that the person watches atleast two hours of television on a selected day?*

(d) *What is the probability that the person watches atmost two hours of television on a selected day?*

(e) *Calculate mathematical expectation.*

(f) *Find the variance and standard deviation of random variable X.*

Solution. From the given information, we find that the probability distribution of X is

X	0	1	2	3
P(X)	0.2	k	$2k$	$2k$

(a) We know that $\Sigma p_i = 1$

$\Rightarrow 0.2 + k + 2k + 2k = 1$

$\Rightarrow 5k = 0.8 \Rightarrow k = \frac{4}{25}.$

(b) Probability that the person watches two hours of television

$$= P(X = 2) = 2k = 2 \times \frac{4}{25} = \frac{8}{25}.$$

(c) P(the person watches atleast two hours of television)

$$= P(X \geq 2) = P(X = 2) + P(X = 3)$$

$$= 2k + 2k = 4k$$

$$= 4 \times \frac{4}{25} = \frac{16}{25}.$$

(d) P(the person watches atmost two hours of television)

$$= P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= 0.2 + k + 2k$$

$$= 0.2 + 3k = \frac{1}{5} + \frac{12}{25} = \frac{17}{25}.$$

(e) We construct the following table:

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	0.2	0	0
1	$\frac{4}{25}$	$\frac{4}{25}$	$\frac{4}{25}$
2	$\frac{8}{25}$	$\frac{16}{25}$	$\frac{32}{25}$
3	$\frac{8}{25}$	$\frac{24}{25}$	$\frac{72}{25}$
Total		$\frac{44}{25}$	$\frac{108}{25}$

$$E(X) = \Sigma p_i x_i = \frac{44}{25} = 1.76$$

(f) Variance $\sigma^2 = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$

$$= \frac{108}{25} - \left(\frac{44}{25}\right)^2 = \frac{108}{25} - \frac{1936}{625} = \frac{764}{625} = 1.22$$

and standard deviation $\sigma = \sqrt{\text{Variance}} = \sqrt{1.22} = 1.1$

Example 10. *An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one by one, at random without replacement. Find the probability distribution of the number of white balls drawn. Also, find the mean and the variance of the number of white balls drawn.*

Solution. Three balls drawn, one by one, without replacement is same as drawing three balls simultaneous.

Total number of balls = 5 + 2 + 3 = 10, 2 balls are white and 8 balls are non-white.

Let X denote the number of white balls drawn in a random draw of three balls, then X can take values 0, 1, 2.

$$P(0) = P\text{ (3 non-white balls)} = \frac{^8C_3}{^{10}C_3} = \frac{8.7.6}{1.2.3} \times \frac{1.2.3}{10.9.8} = \frac{7}{15},$$

$$P(1) = P\text{ (1 white, 2 non-white)} = \frac{{}^2C_1 \times {}^8C_2}{{}^{10}C_3} = 2 \times \frac{8.7}{1.2} \times \frac{1.2.3}{10.9.8} = \frac{7}{15},$$

$$P(2) = P\text{ (2 white, 1 non-white)} = \frac{{}^2C_2 \times {}^8C_1}{{}^{10}C_3} = 1 \times 8 \times \frac{1.2.3}{10.9.8} = \frac{1}{15}.$$

$\therefore$ The probability distribution of white balls is $\begin{pmatrix} 0 & 1 & 2 \\ \frac{7}{15} & \frac{7}{15} & \frac{1}{15} \end{pmatrix}$.

$$\text{Mean} = \mu = \Sigma p_i x_i = \frac{7}{15} \times 0 + \frac{7}{15} \times 1 + \frac{1}{15} \times 2 = \frac{9}{15} = \frac{3}{5}.$$

$$\Sigma p_i x_i^2 = \frac{7}{15} \times 0^2 + \frac{7}{15} \times 1^2 + \frac{1}{15} \times 2^2 = 0 + \frac{7}{15} + \frac{4}{15} = \frac{11}{15}.$$

$$\text{Variance} = \Sigma p_i x_i^2 - \mu^2 = \frac{11}{15} - \left(\frac{3}{5}\right)^2 = \frac{11}{15} - \frac{9}{25} = \frac{55-27}{75} = \frac{28}{75}.$$

Example 11. *In a game, a man wins ₹ 5 for getting a number greater than 4 and loses ₹ 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.*

Solution. Let X (in ₹) denote the amount which the man wins or loses.

As the man throws a die atmost thrice and quits the game as and when he gets a number greater than 4, the sample space of the random experiment is

S = {W, LW, LLW, LLL} where W denotes win and L denotes loss.

The possible values of X are:

X = 5 when he gets a number greater than 4 in the first throw of die,

X = 4 when he gets a number greater than 4 in the second throw of die,

X = 3 when he gets a number greater than 4 in the third throw of die,

X = –3 when he does not get a number greater than 4 in any of three throws of die.

Here, –3 indicates the loss of ₹3.

$$P(\text{getting a number greater than 4}) = P(\text{getting 5 or 6}) = \frac{2}{6} = \frac{1}{3},$$

$$\therefore \quad P(\text{not getting a number greater than 4}) = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$P(X = 5) = \frac{1}{3}, P(X = 4) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9},$$

$$P(X = 3) = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}, P(X = -3) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}.$$

$$\therefore \quad E(X) = \Sigma p_i x_i = \frac{1}{3} \times 5 + \frac{2}{9} \times 4 + \frac{4}{27} \times 3 + \frac{8}{27} \times (-3)$$

$$= \frac{5}{3} + \frac{8}{9} + \frac{4}{9} - \frac{8}{9} = \frac{19}{9}.$$

$\therefore$ The expected amount he wins = ₹ $\frac{19}{9}$ = ₹ $2\frac{1}{9}$.

Example 12. *Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of X. Find the mean and the variance of this distribution.*

Solution. The number of ways of choosing two integers (without replacement) from the first six positive integers = $^6C_2 = 15$, so the sample space S has 15 equally likely outcomes. These outcomes are:

1, 2; 1, 3; 1, 4; 1, 5; 1, 6; 2, 3; 2, 4; 2, 5;

2, 6; 3, 4; 3, 5; 3, 6; 4, 5; 4, 6; 5, 6.

As the random variable X denotes the larger of the two numbers, X can take values 2, 3, 4, 5, 6.

(Because 1 is not larger than any number from 1 to 6)

Note that in the sample space S, we have

Larger of two numbers	Number of outcomes
2	1
3	2
4	3
5	4
6	5

$P(X = 2) = \frac{1}{15}$, $P(X = 3) = \frac{2}{15}$, $P(X = 4) = \frac{3}{15}$,

$P(X = 5) = \frac{4}{15}$, $P(X = 6) = \frac{5}{15}$.

$\therefore$ The probability distribution of X is:

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

$$\therefore \quad \text{Mean} = \mu = \Sigma p_i x_i = \frac{1}{15}(1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6)$$

$$= \frac{1}{15}(2 + 6 + 12 + 20 + 30) = \frac{70}{15} = \frac{14}{3}.$$

$$\text{Now } \Sigma p_i x_i^2 = \frac{1}{15}(1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + 4 \times 5^2 + 5 \times 6^2)$$

$$= \frac{1}{15}(4 + 18 + 48 + 100 + 180) = \frac{350}{15} = \frac{70}{3}.$$

$$\therefore \quad \text{Variance} = \Sigma p_i x_i^2 - \mu^2 = \frac{70}{3} - \left(\frac{14}{3}\right)^2 = \frac{70}{3} - \frac{196}{9} = \frac{14}{9}.$$

EXERCISE 9.2

1. A boy throws a coin. He is to receive two rupees for getting a tail and ten rupees for getting a head. Find his expectation (mean receipt).
2. A game at a school fair costs ₹10, with a prize of ₹50. If the probability of winning is $\frac{1}{10}$, is it a fair game?
3. In a certain village, families are strictly limited to two children. The probability distribution of the number of children of any given individual is as follows :

Number of children	0	1	2
Probability	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{2}{5}$

Find the mean number of children.

4. Find the mean μ and variance σ^2 for the following probability distribution :

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

5. The probability distribution of a random variable X is given as below :

X	0.5	1	1.5	2
P(X)	k	k^2	$2k^2$	k

(i) Find the value of k.

(ii) Determine the mean of the distribution.

6. A discrete random variable X has the following probability distribution :

X	0	1	2	3	4	5
P(X)	$4C^2$	$3C^2$	$2C^2$	C^2	C	2C

(i) Find the value of C.

(ii) Find the mean of the distribution.

(iii) If $\Sigma p_i x_i^2 = 14$, find the variance of the distribution.

7. Find the mean and variance of number of heads in three tosses of a fair coin.
8. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.
9. An urn contains 5 red and 2 black balls. Two balls are randomly drawn without replacement. Find the probability distribution of the black balls drawn. Also, find the mean and variance of the black balls drawn.
10. A box contains 4 red and 5 black marbles. Find the probability distribution of the red marbles in a random draw of three marbles. Also find the mean, variance and standard deviation of the distribution.
11. From a lot of 6 items containing 2 defective items, a sample of 4 items is drawn at random (without replacement). If the random variable X denotes the number of defective items in the sample, find:

(i) the probability distribution of X.

(ii) the mean of the distribution.

(iii) the variance of the distribution.

12. Two positive numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and the variance of the distribution.

Answers

1. ₹ 6

2. No

3. 1.3

4. $\mu = 1.2, \sigma^2 = 0.56$

5. (*i*) $\frac{1}{3}$ (*ii*) $\frac{23}{18}$

6. (*i*) $\frac{1}{5}$ (*ii*) $\frac{16}{5}$ (*iii*) $\frac{94}{25}$

7. $\frac{3}{2}; \frac{3}{4}$

8. $\frac{2}{13}; \frac{400}{2873}, \frac{20\sqrt{17}}{221}$

9. $\begin{pmatrix} 0 & 1 & 2 \\ \frac{10}{21} & \frac{10}{21} & \frac{1}{21} \end{pmatrix}; \frac{4}{7}, \frac{50}{147}$

10. $\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{5}{42} & \frac{20}{42} & \frac{15}{42} & \frac{2}{42} \end{pmatrix}; \frac{4}{3}, \frac{5}{9}, \frac{\sqrt{5}}{3}$

11. (*i*) $\begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{15} & \frac{8}{15} & \frac{6}{15} \end{pmatrix}$ (*ii*) $\frac{4}{3}$ (*iii*) $\frac{16}{45}$

12. Mean = 4, variance = 1